

The Crystal Structure of Bisthiourea-zinc Acetate

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Bisthiourea-zinc acetate, $\text{Zn}[\text{SC}(\text{NH}_2)_2]_2(\text{CH}_3\text{COO})_2$, is monoclinic, $P2_1/c$: $a=6.938$, $b=17.678$, $c=11.795$ Å, $\beta=112.6^\circ$, $Z=4$. The structure, solved by means of three-dimensional Fourier methods, has been refined with the use of the anisotropic differential synthesis. Zn coordinates to two S atoms from two thiourea molecules (Zn-S 2.32₆ and 2.26₁ Å) and to two O atoms (Zn-O 1.97₃ and 1.95₄ Å) from two acetate groups, to form a tetrahedral arrangement. Two further oxygen atoms from the same acetate groups are involved in weaker interactions with the metal atom at 2.99₆ and 2.89₁ Å; as a consequence of these interactions the planes of the acetate ions are oriented in such a way that the Zn atom almost lies within them. There are no significant differences between corresponding bond distances and angles in the two thiourea molecules: these are much the same as in the uncoordinated thiourea.

Introduction

Thiourea adducts with acetates of divalent metals have been prepared and described by Nardelli & Chierici (1959), who showed that the bisthiourea compounds of cobalt and zinc are isostructural. A three-dimensional X-ray analysis has now been carried out to study the coordination of the thiourea molecule and carboxyl group by the metal atom in these compounds. In particular, it seemed interesting to determine the role played by the oxygen atoms in the coordination and the influence, if any, of the sulphur-metal interaction on bond distances and angles in the thiourea molecule.

Experimental

The unit-cell constants, remeasured on single-crystal rotation and Weissenberg photographs and refined by a least-squares procedure on powder diffractometer data (Cu $K\alpha$, $\lambda=1.5418$ Å), are as follows:

$\text{Zn}[\text{SC}(\text{NH}_2)_2]_2(\text{CH}_3\text{COO})_2$; $M=335.7$
 $a=6.938 \pm 0.003$, $b=17.678 \pm 0.006$, $c=11.795 \pm 0.006$ Å
 $\beta=112^\circ 47' \pm 18'$; $V=1334$ Å³; $Z=4$; $D_x=1.671$,
 $D_m=1.61$ g.cm⁻³
 $\mu=54.6$ cm⁻¹ (Cu $K\alpha$), $F(000)=632$.

Space group: $P2_1/c$ (from systematic absences).

Two series of equi-inclination Weissenberg photographs (filtered Cu radiation, multiple film technique) were taken at room temperature about [100] with levels $h=0,1,\dots,6$, and about [001] with levels $l=0,1,\dots,10$. 2128 independent reflexions were observed out of a possible 2554. The intensities were measured photometrically and corrected for Lorentz, polarization and spot shape effects. The sample used to take the photographs around [100] was a prism elongated along this axis and was considered as a cylinder with a mean radius of 0.008 cm in calculating the absorption correction; the photographs around [001] were taken with a nearly spherical fragment with a mean radius of

Table 1. Final atomic fractional coordinates ($\times 10^4$), thermal parameters ($\times 10$ Å²) with e.s.d.'s and ratios (e.s.d.)/(coordinate shift)

	x/a (σ)	y/b (σ)	z/c (σ)	B_{11} (σ)	B_{22} (σ)	B_{33} (σ)	B_{12} (σ)	B_{13} (σ)	B_{23} (σ)	$ r(x) $	$ r(y) $	$ r(z) $
Zn	1938 (2)	1748 (0)	123 (1)	38 (1)	38 (0)	43 (1)	2 (1)	13 (1)	-1 (1)	17	1	11
S(1)	-9 (3)	894 (1)	-1387 (2)	34 (1)	34 (0)	46 (2)	2 (1)	9 (2)	-6 (1)	7	4	8
S(2)	5050 (3)	2063 (1)	54 (2)	36 (2)	45 (0)	33 (2)	-3 (1)	12 (2)	-2 (1)	8	1	∞
O(1)	-117 (10)	2580 (2)	-358 (6)	43 (5)	36 (1)	39 (5)	8 (3)	7 (6)	-4 (4)	12	3	7
O(2)	1838 (10)	3274 (3)	1204 (6)	36 (5)	45 (1)	50 (5)	5 (4)	7 (6)	-13 (4)	5	9	8
O(3)	1827 (11)	1441 (3)	1688 (8)	40 (6)	46 (1)	51 (7)	9 (5)	17 (8)	6 (5)	26	4	6
O(4)	4368 (15)	663 (3)	1952 (10)	55 (10)	50 (2)	64 (10)	18 (8)	29 (12)	13 (8)	9	11	96
N(1)	2758 (16)	448 (3)	-2239 (10)	59 (9)	44 (0)	53 (9)	13 (4)	25 (11)	8 (4)	41	14	50
N(2)	2506 (14)	-278 (2)	-712 (9)	53 (7)	39 (0)	52 (8)	10 (4)	15 (10)	4 (5)	19	∞	9
N(3)	5719 (14)	2607 (4)	2286 (9)	38 (6)	57 (2)	41 (7)	1 (6)	14 (9)	-12 (6)	29	21	5
N(4)	8481 (11)	2110 (4)	2023 (7)	31 (5)	58 (2)	46 (7)	2 (5)	9 (8)	-6 (6)	107	13	9
C(1)	1930 (12)	313 (4)	-1437 (8)	36 (6)	30 (1)	40 (7)	0 (5)	10 (8)	-4 (5)	16	2	8
C(2)	6491 (13)	2277 (3)	1573 (8)	33 (6)	39 (1)	30 (6)	1 (5)	12 (8)	-3 (5)	5	4	4
C(3)	2967 (21)	716 (6)	3481 (13)	57 (11)	66 (3)	49 (10)	1 (9)	19 (14)	5 (9)	8	6	131
C(4)	3080 (13)	944 (4)	2306 (9)	32 (6)	40 (1)	38 (7)	3 (5)	13 (8)	0 (5)	5	7	44
C(5)	-1379 (18)	3778 (5)	-68 (10)	56 (11)	41 (1)	66 (10)	17 (8)	13 (14)	-2 (8)	15	47	98
C(6)	247 (12)	3177 (3)	298 (9)	37 (7)	33 (1)	48 (8)	3 (5)	14 (9)	0 (5)	8	3	17

0.02 cm, and for this series of data the spherical absorption correction was used. The data of both zones were correlated and put on a common scale with the use of the least-squares procedure of Rollett & Sparks (1960). The absolute scale was determined first by Wilson's method, then by comparison with the calculated values.

Structure determination and refinement

The coordinates of Zn, S(1) and S(2) were found first from a three-dimensional Patterson synthesis. The next step was a three-dimensional Fourier synthesis calculated with the use of the signs of the contributions of these atoms to the structure factors ($R=35\%$); this synthesis showed all the other non-hydrogen atoms to be well resolved. Two more three-dimensional Fourier calculations followed by an F_o-F_c synthesis improved the agreement index to 13.1%. The refinement was then carried out by means of nine cycles of Booth's differential synthesis, two calculated with isotropic and seven with anisotropic thermal parameters. The final agreement indices were (R , for observed reflexions only; R' including unobserved reflexions; multiplicities not considered): $R=8.4\%$, $R'=9.2\%$, and the ratios between the e.s.d.'s and the shifts of the coordinates were as shown in Table 1, in which the final parameters

with their e.s.d.'s (Cruickshank, 1949 & 1956) are also quoted. The B_{ij} 's were determined by the method of Nardelli & Fava (1960) using the second derivatives of the electron density in the differential synthesis.

No attempt was made to locate the hydrogen atoms directly. The hydrogen coordinates reported in Table 2, relating to the hydrogen atoms of the two thiourea molecules, were calculated assuming an sp^2 bond configuration for each nitrogen atom with a distance N-H = 1.03 Å. In Table 2 are reported also the values of the electron density found from a final F_o-F_c synthesis at the points corresponding to the assumed coordinates of the hydrogen atoms. Data for the peaks of the other atoms are given in Table 3. Observed and calculated

Table 2. Calculated fractional coordinates and corresponding q_0 values for hydrogen atoms in thiourea molecules

	x/a	y/b	z/c	q_0 ($e.\text{Å}^{-3}$)
H(1)	0.4018	0.0105	-0.2268	1.0
H(2)	0.2368	0.0908	-0.2814	0.7
H(3)	0.1904	-0.0389	-0.0091	0.8
H(4)	0.3759	-0.0622	-0.0744	0.5
H(5)	0.6832	0.2725	0.3186	0.7
H(6)	0.4329	0.2738	0.1950	0.8
H(7)	0.9407	0.2265	0.2908	0.8
H(8)	0.9157	0.1883	0.1461	0.7

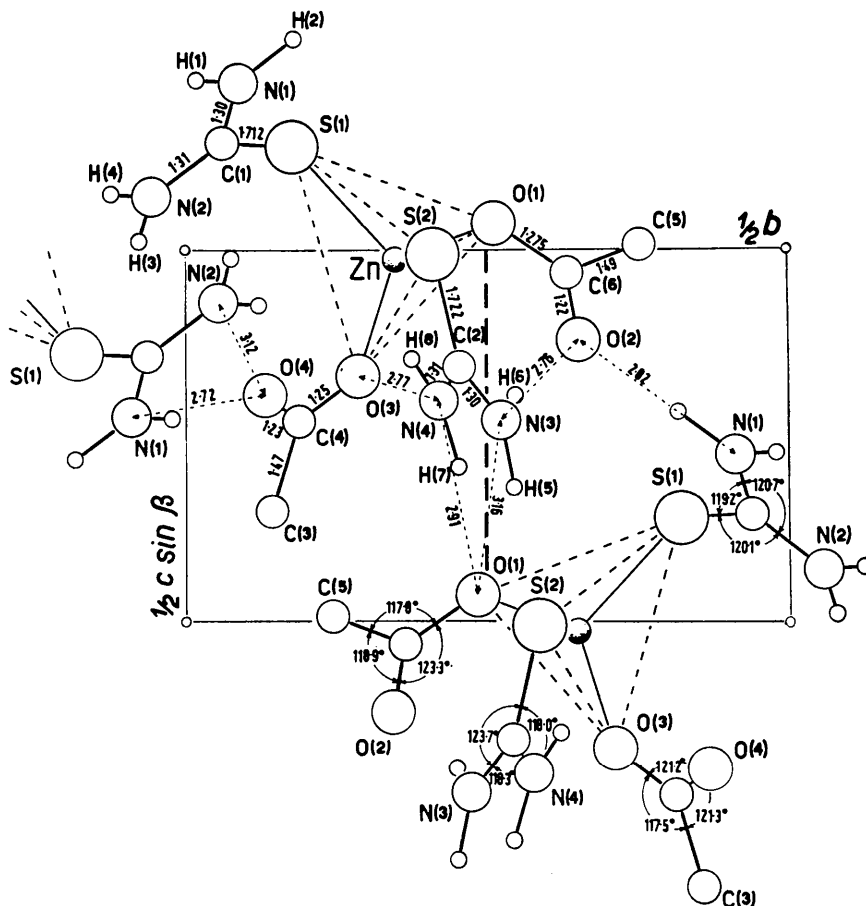


Fig. 1. $Zntu_2(CH_3CO_2)_2$: Diagrammatic projection of the structure along [100].

(not including H atoms) structure factors are compared in Table 4.

The atomic scattering factors used throughout the calculations are those of Thomas & Umeda (1957) for Zn^{2+} , of Dawson (1960) for S and of Berghuis, Haanappel, Potters, Loopstra, MacGillavry & Veenendaal (1955) for O, N and C.

All the calculations were performed on the Olivetti Elea 6001/S computer of the Centro di Calcolo Elettronico della Università di Parma with the programs of Nardelli, Musatti, Domiano & Andreotti (1964, 1965).

Discussion

Fig. 1 shows a diagrammatic projection of the structure along [100]. Each zinc atom coordinates to two sulphur atoms from two thiourea molecules and to two oxygen atoms from two acetate groups, forming a tetrahedral arrangement. Two more oxygen atoms from the same carboxyl groups are involved in a weaker interaction

with the metal atom as shown in Fig. 2. Distances and angles in the coordination polyhedron are (the e.s.d.'s quoted in parentheses are in units of the last place):

Zn-S(1)	2.326(2) Å	Zn-S(2)	2.261(4) Å
Zn-O(1)	1.973(6)	Zn-O(3)	1.954(8)
Zn-O(2)	2.996(5)	Zn-O(4)	2.891(9)

S(1)-Zn-S(2)	114.3°(0.1°)
S(1)-Zn-O(1)	97.1 (0.2)
S(1)-Zn-O(3)	108.6 (0.2)
S(2)-Zn-O(1)	113.6 (0.2)
S(2)-Zn-O(3)	119.3 (0.3)
O(1)-Zn-O(3)	101.0 (0.3)

The two Zn-S distances are significantly different ($t_o = 14.5$, significance test of Cruickshank & Robertson, 1953): one is amongst the shortest hitherto observed [2.286(6) and 2.298(6) Å in tetrahedral monothiosemicarbazide-zinc chloride (Cavalca, Nardelli & Branchi, 1960)]; the other is comparable with the sum of Paul-

Table 3. Atomic peak heights ($e.\text{Å}^{-3}$), curvatures ($e.\text{Å}^{-5}$) and e.s.d.'s

		ρ	$-A_{hh}$	$-A_{kk}$	$-A_{ll}$	A_{kl}	A_{hl}	A_{hk}
Zn	obs.	51.1	429	508	384	-16	117	13
	calc.	53.1	432	518	387	-16	114	13
S(1)	obs.	25.0	217	247	176	3	52	-10
	calc.	26.1	219	253	179	2	51	-10
S(2)	obs.	25.8	223	243	215	0	63	-14
	calc.	26.6	224	250	215	1	59	-15
O(1)	obs.	10.1	79	97	67	0	18	-16
	calc.	10.4	81	98	68	0	18	-17
O(2)	obs.	9.8	73	75	67	-8	18	8
	calc.	10.1	73	77	68	-8	18	8
O(3)	obs.	9.3	69	78	51	-2	16	2
	calc.	9.7	70	80	52	-2	16	2
O(4)	obs.	8.2	64	65	52	0	24	9
	calc.	8.5	65	67	53	-1	24	9
N(1)	obs.	7.2	50	57	45	-7	18	-3
	calc.	7.3	50	58	45	-7	18	-3
N(2)	obs.	7.2	51	68	42	-5	12	-5
	calc.	7.5	51	70	43	-5	12	-4
N(3)	obs.	7.8	53	50	48	1	16	-3
	calc.	8.1	53	51	49	1	15	-3
N(4)	obs.	7.6	65	50	51	-1	13	-5
	calc.	8.0	65	51	52	0	13	-5
C(1)	obs.	7.2	60	58	56	-3	17	4
	calc.	7.6	61	59	57	-3	18	4
C(2)	obs.	7.4	59	62	58	-1	20	-5
	calc.	8.0	59	63	59	-1	19	-5
C(3)	obs.	5.7	41	39	38	1	15	-3
	calc.	5.6	42	40	37	1	15	-3
C(4)	obs.	7.0	58	62	48	-2	15	5
	calc.	7.3	59	64	48	-2	15	6
C(5)	obs.	5.8	35	41	36	-5	7	2
	calc.	6.0	36	42	37	-5	8	2
C(6)	obs.	7.3	57	78	44	-3	12	4
	calc.	7.7	57	79	45	-3	12	4
	e.s.d.	0.2	1	1	1	1	1	1

Table 4 (cont.)

Table with 12 columns (b, k, |1OPo|, 1OPc, h, k, |1|OPo|, 1OPc, h, k, |1|OPo|, 1OPc, h, k, |1|OPo|, 1OPc, h, k, |1|OPo|, 1OPc) and 12 rows of numerical data. The data includes various values such as 2, 3, 6, 146, -152, 5, 10, 62, 42, 44, 1, 1, 5, 536, 472, 2, 9, 7, 259, 260, 2, 19, 7, 42, 44, etc.

important in determining the orientation of the acetic groups.

The two thiourea molecules are planar, their least-squares planes being:*

$$\text{S(1)C(1)N(1)N(2):} \\ 0.4545x' + 0.5720y' + 0.6828z' = 0.1583$$

$$\text{S(2)C(2)N(3)N(4):} \\ 0.3384x' + 0.8984y' - 0.2799z' = 4.4709$$

These two molecules are not significantly different as far as bond distances and angles are concerned. Neither are they significantly different from uncoordinated thiourea (Kunchur & Truter, 1958a, K. & T.), and so it appears that in the present case coordination does not influence the dimensions of the ligand molecule:

K. & T.

$$\begin{array}{l} \text{S(1)-C(1) } 1.712(9) \text{ \AA} \quad \text{S(2)-C(2) } 1.722(9) \text{ \AA} \quad 1.707(12) \\ \text{C(1)-N(1) } 1.305(14) \quad \text{C(2)-N(3) } 1.297(13) \\ \text{C(1)-N(2) } 1.311(10) \quad \text{C(2)-N(4) } 1.307(13) \end{array} \left. \vphantom{\begin{array}{l} \text{S(1)-C(1) } 1.712(9) \text{ \AA} \\ \text{C(1)-N(1) } 1.305(14) \\ \text{C(1)-N(2) } 1.311(10) \end{array}} \right\} 1.311(12)$$

$$\begin{array}{l} \text{N(1)-C(1)-N(2) } 120.7^\circ(0.8^\circ) \\ \text{N(3)-C(2)-N(4) } 118.3(0.8) \\ \text{S(1)-C(1)-N(1) } 119.2(0.6) \\ \text{S(2)-C(2)-N(4) } 118.0(0.7) \\ \text{S(1)-C(1)-N(2) } 120.1(0.7) \\ \text{S(2)-C(2)-N(3) } 123.7(0.8) \end{array} \left. \vphantom{\begin{array}{l} \text{N(1)-C(1)-N(2) } 120.7^\circ(0.8^\circ) \\ \text{N(3)-C(2)-N(4) } 118.3(0.8) \\ \text{S(1)-C(1)-N(1) } 119.2(0.6) \end{array}} \right\} \begin{array}{l} 115.6^\circ(1.1^\circ) \\ 122.2(0.6) \end{array}$$

Both thiourea molecules are tilted with respect to the Zn-S bonds by the same angles: Zn-S(1)-C(1) 100.6°(0.3°), Zn-S(2)-C(2) 101.2°(0.3°). These angles are less than the tetrahedral value for the bonds involving sulphur in thiourea, as observed in other tetrahedral thiourea-metal complexes: *e.g.* 113° in bis-thiourea-cadmium chloride (Nardelli, Cavalca & Braibanti, 1957), 108.6° in bis-thiourea-zinc chloride (Kunchur & Truter, 1958b), 105°, 105°, 108°, 113° in trithiourea-copper(I) chloride (Okaya & Knobler, 1964).

The two acetic ions are also planar, their least-squares planes being:

$$\text{O(1)O(2)C(5)C(6)} \\ 0.6906x' + 0.4243y' - 0.5857z' = 2.2176$$

$$\text{O(3)O(4)C(3)C(4)} \\ 0.5045x' + 0.7212y' + 0.4747z' = 2.9782$$

There are no significant differences between corresponding bonds and angles:

$$\begin{array}{ll} \text{C(3)-C(4) } 1.474(17) \text{ \AA} & \text{C(5)-C(6) } 1.487(13) \text{ \AA} \\ \text{C(4)-O(4) } 1.228(14) & \text{C(6)-O(2) } 1.215(11) \\ \text{C(4)-O(3) } 1.252(11) & \text{C(6)-O(1) } 1.275(8) \end{array}$$

$$\begin{array}{ll} \text{O(3)-C(4)-O(4) } 121.2^\circ(0.9^\circ) \\ \text{O(1)-C(6)-O(2) } 123.3(0.7) \\ \text{C(3)-C(4)-O(4) } 121.3(0.9) \\ \text{C(5)-C(6)-O(2) } 118.9(0.7) \\ \text{C(3)-C(4)-O(3) } 117.5(0.9) \\ \text{C(5)-C(6)-O(1) } 117.8(0.8) \end{array}$$

In both these two ions the C-O bond, involving the oxygen atom more closely linked to the metal atom, is longer than that involving the oxygen atom belonging to the second coordination sphere. Consequently, there is an asymmetry of the carboxyl group, which is also observed in the free acids (Nardelli, Fava & Giraldi, 1962) and in several metal compounds; *e.g.* 1.275(14) and 1.226(15) Å, 1.291(15) and 1.243(15) Å in bisglycino-copper(II) monohydrate (Freeman, Snow, Nitta & Tomita, 1964), 1.311(11) and 1.206(12) Å in glycyglycylglycinecopper(II) chloride sesquihydrate (Freeman, Robinson & Schoone, 1964), 1.28(1) and 1.23(1) Å in sodium glycyglycylglycinecuprate(II) monohydrate (Freeman, Schoone & Sime, 1965).

The orientations of the planes of the two acetate ions are determined by the interactions Zn-O(1), Zn-O(2) and Zn-O(3), Zn-O(4) and by the trigonal character of the bonds formed by the oxygen atoms and implied by the shortest contacts: Zn-O(1)-C(6) 119.4°(0.6°), Zn-O(3)-C(4) 118.2°(0.7°). It follows that the Zn atom lies near to the two acetate planes: the distances are 0.095 Å from the O(1)O(2)C(5)C(6) plane and 0.036 Å from the O(3)O(4)C(3)C(4) plane. The longer is this distance, the weaker is the long-range Zn-O interaction.

The following N...O distances must be considered as hydrogen bonds (the corresponding H-N-O angles are quoted in square brackets; the *e.s.d.*'s are all 0.01 Å):

$$\begin{array}{ll} \text{N(1)-H(2)...O(2}^{vi}) & 2.82 \text{ \AA} [4.7^\circ] \\ \text{N(1)-H(1)...O(4}^{vii}) & 2.72 [14.1] \\ \text{N(3)-H(6)...O(2)} & 2.76 [11.3] \\ \text{N(4)-H(7)...O(1}^{viii}) & 2.91 [18.7] \\ \text{N(4)-H(8)...O(3}^{v}) & 2.77 [29.0] \end{array}$$

The packing in the crystal and the orientation of the thiourea molecules are determined by these interactions. Other distances less than 3.5 Å are as follows:

$$\begin{array}{llll} \text{S(1)-O(1)} & 3.23 \text{ \AA} & \text{O(3)-C(6)} & 3.45 \text{ \AA} \\ \text{S(1)-O(3)} & 3.48 & \text{O(3)-N(2}^{iv}) & 3.45 \\ \text{S(2)-O(4)} & 3.49 & \text{O(4)-N(2)} & 3.34 \\ \text{O(1)-N(4}^{\prime}) & 3.41 & \text{O(4)-N(2}^{vii}) & 3.12 \\ \text{O(1)-N(3}^{viii}) & 3.16 & \text{O(4)-C(2)} & 3.32 \\ \text{O(1)-O(3)} & 3.03 & \text{O(4)-C(1}^{v}) & 3.34 \\ \text{O(1)-C(2}^{viii}) & 3.48 & \text{N(3)-C(4)} & 3.47 \\ \text{O(2)-O(3)} & 3.29 & \text{N(4)-C(6}^{\prime\prime}) & 3.34 \\ \text{O(3)-N(3)} & 3.25 & & \end{array}$$

* Transformation matrix from monoclinic *x, y, z* to orthogonal *x', y', z'* coordinates:

$$\begin{pmatrix} 1 & 0 & \cos \beta \\ 0 & 1 & 0 \\ 0 & 0 & \sin \beta \end{pmatrix}$$

$$\begin{array}{ll} \prime & x-1, y, z \\ \prime\prime & x+1, y, z \\ \prime\prime\prime & 1-x, \bar{y}, \bar{z} \\ iv & \bar{x}, \bar{y}, z \\ v & x-1, \frac{1}{2}-y, z+\frac{1}{2} \\ vi & x, \frac{1}{2}-y, z-\frac{1}{2} \\ vii & x+1, \frac{1}{2}-y, z+\frac{1}{2} \\ viii & x-1, \frac{1}{2}-y, z-\frac{1}{2} \end{array}$$

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References

- BERGHUIS, J., HAANAPPEL, IJ. M., POTTERS, M., LOOPSTRA, B. O., MACGILLAVRY, C. H. & VEENENDAAL, A. L. (1955). *Acta Cryst.* **8**, 478.
- BONAMICO, M., MAZZONE, G., VACIAGO, A. & ZAMBONELLI, L. (1965). *Acta Cryst.* **19**, 898.
- CAVALCA, L., NARDELLI, M. & BRANCHI, G. (1960). *Acta Cryst.* **13**, 688.
- COTTON, E. A. & WOOD, J. S. (1964). *Inorg. Chem.* **3**, 245.
- CRUICKSHANK, D. W. J. (1949). *Acta Cryst.* **2**, 65.
- CRUICKSHANK, D. W. J. (1956). *Acta Cryst.* **9**, 747.
- CRUICKSHANK, D. W. J. & ROBERTSON, A. P. (1953). *Acta Cryst.* **6**, 698.
- DAWSON, B. (1960). *Acta Cryst.* **13**, 403.
- FREEMAN, H. C., ROBINSON, G. & SCHOONE, J. C. (1964). *Acta Cryst.* **17**, 719.
- FREEMAN, H. C., SCHOONE, J. C. & SIME, J. G. (1965). *Acta Cryst.* **18**, 381.
- FREEMAN, H. C., SNOW, M. R., NITTA, I. & TOMITA, K. (1964). *Acta Cryst.* **17**, 1463.
- HARDING, M. M. & COLE, S. J. (1963). *Acta Cryst.* **16**, 643.
- KLUG, H. P., ALEXANDER, L. E. & SUMNER, G. G. (1958). *Acta Cryst.* **11**, 41.
- KRETSINGER, R. H., COTTON, F. A. & BRYAN, R. F. (1963). *Acta Cryst.* **16**, 651.
- KUNCHUR, N. R. & TRUTER, M. R. (1958a). *J. Chem. Soc.* p. 2551.
- KUNCHUR, N. R. & TRUTER, M. R. (1958b). *J. Chem. Soc.* p. 3478.
- NARDELLI, M., CAVALCA, L. & BRAIBANTI, A. (1957). *Gazz. chim. Ital.* **87**, 137.
- NARDELLI, M. & CHIERICI, I. (1959). *Ric. Sci.* **29**, 1731.
- NARDELLI, M. & FAVA, G. (1960). *Ric. Sci.* **30**, 898.
- NARDELLI, M., FAVA, G. & GIRALDI, G. (1962). *Acta Cryst.* **15**, 737.
- NARDELLI, M., MUSATTI, A., DOMIANO, P. & ANDRETTI, G. (1964). *Ric. Sci.* **34**, II-A, 711.
- NARDELLI, M., MUSATTI, A., DOMIANO, P. & ANDRETTI, G. (1965). *Ric. Sci.* **35**, II-A, 469, 477 & 807.
- OKAYA, Y. & KNOBLER, C. B. (1964). *Acta Cryst.* **17**, 928.
- ROLLETT, J. S. & SPARKS, R. A. (1960). *Acta Cryst.* **13**, 263.
- THOMAS, L. H. & UMEDA, K. (1957). *J. Chem. Phys.* **26**, 293.

Acta Cryst. (1967). **22**, 98

Expectation Value of the Bijvoet Ratio*

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The theoretical expressions for the expectation values of the Bijvoet difference and the Bijvoet ratio have been derived for a non-centrosymmetric crystal in which the anomalous scatterers are all of the same type. The dependence of the expectation value of the Bijvoet ratio on the number and the strength of the anomalous scatterers in the unit cell is discussed and this is used to obtain the best conditions for accurate measurement of the Bijvoet differences of a fairly large percentage of reflexions.

1. Introduction

The increasing importance of the anomalous dispersion method in crystal structure analysis necessitates a theoretical study of the measurability of Bijvoet differences. One approach to this would be a study of the statistical distribution of Bijvoet differences, ΔI . The distribution of Bijvoet differences in a normalized form, viz. $x = |\Delta I|/4[\langle I_Q \rangle \langle I_P'' \rangle]^{\frac{1}{2}}$ (where P and Q refer to the anomalous and normal scatterers respectively) has already been worked out (Parthasarathy & Srinivasan, 1964). However, the distribution of ΔI normalized by the local mean intensity, $\langle I_N \rangle$ (where $N = P + Q$) rather than by the factor $4[\langle I_Q \rangle \langle I_P'' \rangle]^{\frac{1}{2}}$ is probably more useful. The distribution of this quantity Δ

($= |\Delta I|/\langle I_N \rangle$) can easily be obtained from the known distribution of x , since we have in the usual notation

$$\Delta = \frac{|\Delta I|}{\langle I_N \rangle} = \frac{|\Delta I|}{4[\langle I_Q \rangle \langle I_P'' \rangle]^{\frac{1}{2}}} \times \frac{4[\langle I_Q \rangle \langle I_P'' \rangle]^{\frac{1}{2}}}{\langle I_N \rangle} = 4k\sigma_1\sigma_2x \quad (1)$$

where $k = f_P''/f_P'$, the ratio of the imaginary to the total real part of the atomic scattering factor of the anomalous scatterer and σ_1^2 and σ_2^2 are the fractional contribution to the mean intensity by the P - and the Q -atoms respectively.

A better normalization factor would be the mean intensity I of the Bijvoet pair of reflexions, i.e. $I = \frac{1}{2}[I(\mathbf{H}) + I(\bar{\mathbf{H}})]$. The normalized Bijvoet difference $|\Delta I|/I$ (denoted by δ) is called the Bijvoet ratio in this paper. The distribution function of δ is difficult to cal-

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